leaving a given type unaltered. The order of the group is not the same for any two types, so that no two are equivalent.

For type A, the group is triply transitive, of order 1344, generated by $S=(257)(346)$, $T=(36)(58)$, $V=(1537)(46)$. Obvious products of these replace 1 by 1, ..., or 8. Now S and V transform T into $R=(34)(78)$ and $P=(47)(38)$. Obvious products of these five leave 1 fixed and replace 2 by 2, ..., 8; others leave 2 and 3 fixed and replace 3 by 3, ..., 8. Now $R^{-1}TR=(46)(57)$, which is transformed into (45)(67) by P. Hence there exist substitutions leaving 1, 2, 3 fixed and replacing 4 by 4, 5, 6, or 7, but not by 8, since 8 is necessarily fixed by the fifth line of A. When 1, 2, 3, 4 are fixed, all are fixed. Hence the order is 8.7.6.4.

For type C, the group is simply transitive, of order 24, generated by $R_1=(126)(385)$, $S_1=(267)(345)$, $T_1=(13)(24)(57)(68)$. There exist substitutions, leaving C unaltered, of the form (1) ($2x$...) for $x=2$, 6, 7, but not for $x=3$, 4, 5, 8. When this is verified for $x=3$ and 8, it follows for $x=4$, 5. Thus, if $W=(1)(24...)$ occurred, then would $WS_1^{-1}=(1)(23...)$ occur. Since identity alone leaves 1 and 2 fixed, the order is 8.3.

For type D, the group is simply transitive, of order 96, generated by (1254)(3678), (13)(57), (24)(68), (537)(648), the first transforming the second into (26)(48). Hence there occur substitutions (1)($2x$...) for $x=2$, 4, 6, 8. I find that no substitution (1)(23...) leaves D unaltered. Hence this is true for (1)(25...) and (1)(27...), in view of the last generator. If, for y even, (1)(2)($3y$...) occurred, then by transforming by (24)(68), (26)(48) or by their product, we would reach (1)(32...), whereas its inverse does not occur. When 1, 2, and 3 are fixed, all are; hence the order is 8.4.3.

For the type E, the group is transitive, of order 64, and is generated by (1472)(3856), (1876)(2345), (17)(68), (15)(37), (17)(35). Combinations of the first two replace 4 by 1, ..., 8. A substitution which leaves 4 fixed must replace 6 by 6 or 8. Now (4)(6)($5x$...) occurs if and only if $x=1$, 3, 5, 7. If 1, 4, 6 are fixed, all are. The order is 8.2.4.

For type F, the group is intransitive, of order 42, and generated by (2345678), (346)(587), (38)(47)(56), being the metacyclic group commutative with the cyclic G_7.

For type B it was found that (28)(46) is the only non-identical substitution not altering B and leaving 1 fixed. Another substitution occurring is (15)(37). Hence the group is of order $4n$, $n=1$, 2, 3, or 4. In any event, the order is less than the orders in the previous cases.

The number of conjugate programmes of type A is $8!\div 1344=15$.

MISCELLANEOUS.

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158. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

An ingot of pure gold was melted at the Mint and then 10 ounces were taken out and 10 ounces of pure silver added and the contents of the melting pot

mixed thoroughly. This was repeated until there were 10 such operations in all. The contents of the pot being then assayed was found to be nine-tenths fine, or standard gold. What was the weight of the original ingot? There was no loss in the precious metals by the melting.

Solution by the PROPOSER.

Let X=the weight of the original pure gold ingot; $n=10$=the number of operations; U_n=the weight of pure gold in the pot after the nth operation; $c=10$=the weight of metal taken from the pot, and also the weight of the silver put in, at each operation; cU_nX^{-1}=the weight of pure gold taken out at the $(n+1)$th operation; U_{n+1}=the weight of the pure gold in the pot after the $(n+1)$th operation; and $U_n=0.9X$.

Equate the elements defined above and we have:

$$U_{n+1}=U_n-cU_nX^{-1}\ldots(1);\text{ or }U_{n+1}=(1-cX^{-1})U_n\ldots(2).$$

This is an equation in Finite Differences. Integrate it, and we have:

$$U_n=C(1-cX^{-1})^n\ldots(3).$$

Equation (3) is true for all values of n. When $n=0$, $U_n=U_0=X$, and $C=X$. Substitute this value of C in (3) and we have:

$$U_n=X(1-cX^{-1})^n\ldots(4).$$

We have $U_n=0.9X$. Eliminate U_n, supply numerical values, reduce, and we have:

$$(1-10X^{-1})^{10}=0.9\ldots(5).$$

Therefore $1-10X^{-1}=\sqrt[10]{(0.9)}$; or $X=10\div[1-\sqrt[10]{(0.9)}]$.

The log of $0.9=\bar{1}.954242509=\overline{10}+9.9542425094$, this divided by 10 gives $\bar{1}.9954242509$, which is the log of 0.9895192581, this subtracted from 1, gives 0.01048007419, the log of which is $\bar{2}.0203920257$, which subtracted from 1 (the log of 10) gives 2.9796079743, which is the log of $954.1309293=X$, the original weight of the pure gold ingot, in ounces.

Also solved by S. A. Corey, G. W. Greenwood, and J. Scheffer.